International Journal of Physics and Mathematical Research
Vol. 3, No. 2, 011-013, February, 2014
http://www.wrpjournals.com/IJPMR

Research Article

Strongly Generalized closed sets in Ideal Topological Spaces

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Received Article 20th January, 2013; Published Article 21st February, 2014

Abstract: In this paper, we define a new class of generalized closed sets namely strongly generalized closed sets in Ideal topological spaces. Also, we study some properties of strongly generalized closed sets with respect to an ideal.

Key words: SI_g – closed set, SI_g – open set.

INTRODUCTION

One of the important tools in General Topology is the Ideals. Newcomb (1967), Rancin (1972), Samuals (1975), Hamlet and Jankovic (1990, 1992, 1990) motivated the research in General topology. A generalized closed set in topological space was introduced by Levine (1967) in 1970. The notion of ideal topological spaces was studied by Kurotowski (1933) and Vaidyanathaswamy (1945). Jafari and Rajesh introduced Ig-closed set with respect to an Ideal and Basari Kodi introduced Is*g-closed sets in Ideal topological spaces. In this paper, we introduce and study a new class of generalized closed sets in Ideal topological spaces called SI_g-closed sets with respect to an Ideal which is the extension of Ig and Is*g closed sets in Ideal topological spaces.

Preliminary Notes

Throughout the present paper (X, τ) always means a topological space. Let A be a subset of a topological space (X, τ). The closure (resp. interior) of A are denoted by Cl(A)(resp. Int(A)). An ideal (Kuratowski, 1933) on a set X is a nonempty collection of subsets of X with heredity property and finite additivity property that is it satisfies the following two conditions:

1. A ∈I and B ⊆A then B ∈I(heredity)
2. A ∈I and B ∈I implies A ∪B ∈I (finite additivity)

Let A ⊆ B ⊆ X. Then $cl_B(A)$(respectively $int_B(A)$)denotes the closure of A(respectively interior of A)with respect to B.

Definition 2.1. A subset A of a topological space (X, τ) is said to be:

1. g- closed [10]if cl(A) ⊆ U whenever A ⊆ U and U is open.
2. s*g- closed [4] if cl(A) ⊆ A whenever A ⊆ U and U is semi-open.

Definition 2. 2[10] Let (X, τ) be a topological space and I be an ideal on X. A subset A of X is said to be generalized closed set with respect to an ideal (briefly Ig- closed) if and only if cl(A) –B ∈I whenever A ⊆B and B is open.

Definition 2. 3 [11] Let (X, τ) be a topological space and I be an ideal on X. A subset A of X is said to be s*g-closed set with respect to an ideal (briefly Is*g-closed) if and only if cl(A)–B ∈I whenever A ⊆B and B is semi-open.

Strongly Generalized closed sets with respect to an Ideal

Definition 3. 1

Let (X, τ) be a topological space and I be an ideal on X. A subset A of X is said to be strongly generalized closed set with respect to an ideal (briefly SI_g- closed) if and only if cl(int(A))–B ∈I whenever A ⊆B and B is open.

Theorem 3. 2

Every g- closed set is a strongly generalized closed set with respect to an ideal but the converse need not be true.

Example 3. 3

Let X = {a, b, c} with τ = { ϕ, {a},{a, c}, X} and I = { ϕ, {b}, {c}, {b, c}}. Here A = {c} is SI_g- closed set but not g-closed set in (X, τ).

Theorem 3.4

Every Is*g-closed set is SI_g-closed but the converse need not be true.

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Proof: Assume that A is Is*g- closed. That is cl(A) – B ∈I whenever A ⊆B and B is semi-open. Let A ⊆U where U is open. This implies that A ⊆ U where U is semi-open. Therefore, cl(A) – B ∈I. This implies cl(int(A)) – B ∈I.

Example 3. 5 Let X = {a, b, c} with τ ={ ϕ, {a},{a, c}, X}and I={ ϕ, {a},{b},{a,b}}. Here A = {a}is a SI_g-closed set but not a Is*g-closed set.

Theorem 3. 6

A set A is SI_g -closed set if and only if F ⊆ cl(int(A)) – A and F is closed in X implies F ∈I.

Proof: Assume that A is Ig-closed. Let F ⊆cl(int(A)) – A. Suppose F is closed. Since F ⊆cl(int(A)) – A , F ⊆X – A and so A ⊆X – F. By our assumption, cl(int(A)) – (X – F) ∈ I. But F ⊆ cl(int(A)) – (X − F) and hence F ∈ I. Conversely, assume that F ⊆cl(int(A)) – A and F is closed in X implies that F ∈I. Suppose A ⊆ U and U is open. Then cl(int(A)) – U = cl(int(A)) ∩ (X–U) is a closed set in X, and is contained in cl(int(A)) – A. By assumption, cl(int(A)) – U ∈I. This implies that A is a SI_g-closed set.

Theorem 3.7

If A is a SI_g-closed subset of (X, τ) and if cl(int(A)) – A contain any closed set in(X, τ) then cl(int(A))∩ F ∈I.

Proof: Let F be a closed set in (X, τ) such that F cl(int(A)) – AThen F ⊆ X – A. Since A is Isg-closed and X – F is open, cl(int(A)) – (X – F) ∈I.

Theorem 3. 8

If A is SI_g-closed and A ⊆B ⊆cl(int(A)) in (X, τ), then B is SI_g-closed in (X, τ).

Proof: Suppose A is SI_g -closed and A ⊆B ⊆cl(int(A)) in (X, τ). Suppose B ⊆U and U is open. Then A ⊆U. Since A is SI_g-closed, we have cl(int(A)) – U ∈ I. Now B ⊆ cl(int(A)). This implies that cl(int(B)) – U cl(int(A)) – U ∈ I. Hence B is SI_g-closed in (X, τ).

Theorem 3. 9

Let A ⊆Y ⊆X and suppose that A is SI_g -closed in (X, τ). Then A is SI_g -closed relative to the subspace Y of X, with respect to the ideal I_Y = {F ⊆Y: F ∈I}.

Proof: Suppose A ⊆U ∩ Y and U is open in (X, τ), then A ⊆U. Since A is SI_g-closed in (X, τ), we have cl(int(A)) – U ∈I. Now (cl((int(A)) ∩ Y) – (U∩Y) = (cl(int(A)) – U)∩Y ∈I, whenever A ⊆U ∩ Y and U is open. Hence A is SI_g-closed relative to the subspace Y.

Theorem 3. 10

If A and B are SI_g-closed sets of (X, τ), then their union A∪ B is also SI_g-closed.

Proof: Suppose A and B are SI_g-closed sets in (X, τ). If A ∪ B ⊆ U and U is open, then A ⊆ U and B ⊆ U. By assumption, cl(int(A)) – U ∈ I and cl(int(B)) – U ∈ I and hence cl(int(A ∪ B)) – U = (cl(int(A))–U) ∪ (cl(int(B))–U) ∈I. That is, A ∪B is SI_g-closed.

Remark 3. 11

The intersection of two SI_g-closed sets need not be an SI_g-closed set.

Theorem 3. 12

The intersection of SI_g-closed set and F be a closed set in (X, τ) is an SI_g-closed set in (X, τ).

Proof: Let A ∩ F ⊆U and U is open. Then A ⊆U ∪(X–F). Since A is SI_g-closed, we have cl(int(A)) – (U ∪(X – F)) ∈I. Now, cl(int(A ∩ F)) ⊆cl(int(A)) ∩ F = (cl(int(A)) ∩ F) – (X–F). Therefore, cl(int(A∩ F)) − U ⊆(cl(int(A)) ∩ F) −(U ∩ (X − F))⊆cl(int(A))−(U ∪(X − F))∈ I
Hence A ∩ F is SI_g -closed in (X, τ).

Definition 3. 13

Let (X, τ) be a topological space and I be an ideal on X. A subset A ⊆X is said to be strongly generalized open with respect to an ideal (briefly SI_g -open) if and only if X–A is SI_g -closed.

Theorem 3. 14

A set A is SI_g-open in (X, τ) if and only if F – U ⊆int(A), for some U ∈I, whenever F ⊆A and F is closed.

Proof: Suppose A is SI_g-open. Suppose F ⊆ A and F is closed. We have X – A ⊆X – F. By assumption, cl(int(X – A)) ⊆(X – F) ∪U, for some U ∈I. This implies X – ((X – F) ∪U) ⊆X – cl(int(X – A)))and hence F – U ⊆int(A).

Conversely, assume that F ⊆A and F is closed implies F – U ⊆int(A), for some U ∈I. Consider an open set G such that X – A ⊆G. Then X – G ⊆A. By assumption, (X – G) – U ⊆int(A) = X – cl(int(X – A)). This gives that X – (G ∪ U) ⊆X – cl(int(X–A)).Then, cl(int((X – A)) ⊆G ∪U, for some U ∈I. This shows that cl(int(X – A)) – G ∈I. Hence X – A is SI_g-closed.

Recall that the sets A and B are said to be separated if cl(A) ∩ B = ϕ and A ∩ cl(B) = ϕ.

Theorem 3. 15

If A and B are separated SI_g-open sets in (X, τ), then A ∪B is SI_g-open.

Proof: Suppose A and B are separated SI_g-open sets in (X, τ) and F be a closed subset of A ∪B.

Then F ∩ cl(int(A)) ⊆ A and F ∩ cl(int(B)) ⊆ B. By assumption and by theorem 3. 14, (F ∩ cl(int(A))) – U_1 ⊆ int(A) and F ∩ cl(int(B))) – U_2 ⊆ int(B), for some U_1, U_2 ∈ I. This mean that ((F ∩ cl(int(A))) – int(A)) ∈ I and (F ∩ cl(int(B))) – int(B) ∈ I. Then ((F ∩ cl(int(A))) – int(A)) ∪ ((F ∩ cl(int(B))) –int(B)) ∈ I. Hence (F ∩ (cl(int(A)) ∪ cl(int(B))) – (int(A) ∪int(B))) ∈ I. But F=F∩(A∪B) ⊂ F∩cl(int(A∪B)), and we have F − int(A∪ B) ⊆ (F ∩ cl(int(A ∪B))) − int(A∪B) ⊆(F ∩ cl(int(A∪B))) −(int(A) ∪ int(B)) ∈I. Hence, F–U ⊆int(A ∪B), for some U ∈I. This proves that A ∪B is SI_g-open.

Theorem 3. 16

If A and B are SI_g-open sets in (X, τ) , then A ∩ B is SI_g-open.

Proof: If A and B are SI_g-open, then X – A and X – B are SI_g-closed. By Theorem 3. 9, X – (A ∩ B) is SI_g-closed, which implies A ∩ B is SI_g-open.

Theorem 3. 17

If int(A) ⊆B ⊆A and if A is SI_g-open in (X, τ), then B is SI_g-open in X.

Proof: Suppose int(A) ⊆B ⊆A and A is SI_g-open. Then X – A ⊆X – B ⊆cl(int(X – A)) and X – A is Ig-closed. By theorem 3. 8, X–B is SI_g -closed and hence B is SI_g-open.

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